Instructors:	Philadelphia University	Academic Year: 2023-2024
1. Dr. Ghada Al-Afifi **2.** Dr. Rola Alseidi **3.** Dr. Abdallah Soboh **4.** Mr. Mohammad Al-Momani	Faculty of Science Department of Mathematics Midterm Exam	**Semester:** Fall **Date:** 16/12/2023 **Course:** Linear Algebra (1) **Duration:** 60 Min.

Name: **I.D. Number:**

Question One: [12 points] **Choose the correct answer and fill your answers in the table provided.**

Question	01	02	03	04	05	06	07	08
Answer								

1. If $\det(A) = 5$ and $\det(B^{-1}) = 2$, then $\det(AB)^{-1} =$

   (A) 10 **(B)** -10 **(C)** $\frac{5}{2}$ **(D)** $\frac{2}{5}$ **(E)** None.

2. Let A be 3×3 matrix with $\det(A) = 2$. Then $\det(2adj(A)) =$

   (A) 16 **(B)** 10 **(C)** 32 **(D)** 8 **(E)** None

3. Let $A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \\ 8 & -1 & 6 \end{bmatrix}$. Then the cofactor $C_{21} =$

   (A) -13 **(B)** 14 **(C)** 13 **(D)** -14 **(E)** None

4. If $A = \begin{bmatrix} 3 & 6 & 1 \\ -3 & 4 & 5 \\ 4 & 2 & 3 \end{bmatrix}$, then $tr(A + A^T) =$

   (A) 5 **(B)** 20 **(C)** 10 **(D)** 18 **(E)** None

5. Let A, X be square matrices of the same size. If X is invertible, then $(XAX^{-1})^4 =$

(A) XA^4X^{-1} **(B)** A^4 **(C)** $X^{-1}A^4X$ **(D)** $X^4A^4X^{-4}$ **(E)** $X^{-4}A^4X^4$

6. The value of k for which the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & k+2 \end{bmatrix}$ is singular is **:**

(A) 2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** -2

7. One of the following matrices is in row-echelon form **:**

(A) $\begin{bmatrix} 1 & 0 & 4 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(E)** None

8. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, then $\begin{vmatrix} -a & -b & -c \\ 2d & 2e & 2f \\ 3a+g & 3b+h & 3c+i \end{vmatrix} =$ **:**

(A) -6 **(B)** -18 **(C)** 6 **(D)** 18 **(E)** None

Question Two:[6 points] Circle True or False. Read each statement carefully before answering.

1. True False The transpose of an upper triangular matrix is an upper triangular matrix.
2. True False For two square matrices A, and B, $\det(A+B) = \det(A) + \det(B)$.
3. True False The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ has no main diagonal.
4. True False For all square matrices A and B of the same size, $(A+B)^2 = A^2 + 2AB + B^2$.
5. True False If A is a singular $n \times n$ matrix, then the system $AX = 0$ has infinitely many solutions.
6. True False Let A be a square matrix, then AA^T is symmetric.

Question Three:[4 points] Let $B = \begin{bmatrix} 1 & 6 \\ 0 & 3 \end{bmatrix}$

1. Find B^{-1}.

2. Find the matrix A such that $(A^T + I_2) = B^{-1}$.

Question Four:[4 points] Use Cramer's rule to solve the system

$$\begin{aligned} 2x - y &= 3 \\ x + 2y &= -1 \end{aligned}$$

Question Five:[4 points] Consider the following linear system

$$\begin{aligned} x + y &= 1 \\ k^2 x + y &= k \end{aligned}$$

Find all values of k such that the system.

1. has infinitely many solutions.

2. has a unique solution.

Question Six:[2 points] If $A^2 + 2A = I$, show that A is invertible.